Green Chicken Contest - 2015

1. The 2015 members of the Blackjack Weapons Company attend a Halloween Party. Among the employees are 5 foreign spies. The F.B.I. knows that the spies will all dress in astronaut costumes. Unfortunately, there are also two other employees dressed the same way. As the employees exit the party in single file, the F.B.I. agents arrest the first five astronauts that they see. What is the probability that they arrest only foreign spies?

2. What is the least positive integer n such that n plus a prime is never a perfect square?

3. Determine the following sums:

   (a) (1 pt.) $\sum_{n=0}^{\infty} \frac{1}{n!}$

   (b) (2 pts.) $\sum_{n=0}^{\infty} \frac{1}{(n+1)n!}$

   (c) (7 pts.) $\sum_{n=0}^{\infty} \frac{1}{(n+2)n!}$

4. Consider the set of circles $\{C(r)\}$ for all r with $0 \le r \le 2$ where C(r) is the circle of radius r centered at (2r, 0). Let P be the set of points contained in C(r) for some r. What is the area of P?

5. Let n be a positive integer. Show that there is a positive multiple of n whose digits consist solely of 0's and 1's.

6. (a) (1 pt.) Verify that if $f(x) = 2\sin(x/2)$ then $|f(x)| \le 2$ and $f(x)f'(x) = \sin x$ for all x.

   (b) (9 pts.) Is there a function f(x), differentiable for all x, such that $|f(x)| < 2$ and $f(x)f'(x) \ge \sin x$ for all x? If so, give an explicit example. If not, prove why not.