Review of "The Magic of Math"

By Gabriel Ngwe

"The Magic of Math" is a book on general mathematics which aims to reveal the underlying magic. What lies behind "The Magic of Math" is the same thing that lies behind magic in general: manipulation and redirection. These themes underlie the book, and Arthur Benjamin uses them well to explain the basic concepts.

One of Benjamin's best examples of the magic in mathematics is infinite series, where evaluating a sum may lead to contradictory results. One example is the "proof" that the sum of all powers of 2 is -1. Let

$$\begin{aligned} S &= 1 + 2 + 4 + 8 + 16 + \cdots \\ &= 1 + 2(1 + 2 + 4 + 8 + 16 + \cdots) \\ &= 1 + 2S, \end{aligned}$$

so $S = -1$.

After introducing the topic of infinite series Benjamin gives an interesting application from probability. Suppose we roll two six-sided dice until we get a 6 or a 7. We want to determine the probability that we get a 6 before a 7. On the first roll, the probability of rolling a 6 is 5/36. In order to roll a 6 before a 7 on the second roll, we must not roll either of them on the first roll and a 6 on the second roll. The probability not rolling a 6 or 7 is

$$1 - \left(\frac{5}{36} + \frac{6}{36}\right) = \frac{25}{36},$$

and the probability of rolling 6 is 5/36, so the probability of rolling a 6 before a 7 on the second roll is (25/36)(5/36). To roll a 6 before a 7 on the third roll, we must not get a 6 or a 7 on the first two rolls and a 6 on the third roll. The probability of not rolling a 6 or 7 on the first two rolls is $(25/36)^2$, and the probability of rolling a 6 on the third roll is (5/36), so the probability of rolling a 6 on the third roll is $(25/36)^2(5/36)$. For the fourth roll, we have a probability of $(25/36)^3(5/36)$, and so on. Summing all of these probabilities gives us

$$\begin{aligned} &\frac{5}{36} + \left(\frac{25}{36}\right)\left(\frac{5}{36}\right) + \left(\frac{25}{36}\right)^2\left(\frac{5}{36}\right) + \cdots \\ &= \tfrac{5}{36}\left[1 + \tfrac{25}{36} + \left(\tfrac{25}{36}\right)^2 + \left(\tfrac{25}{36}\right)^3 + \cdots\right] \\ &= \frac{5}{36}\left(\frac{1}{1 - \left(\frac{25}{36}\right)}\right) = \frac{5}{11}, \end{aligned}$$

the probability of rolling a six before a seven. Much of the book follows this pattern: statement of the result, explanation and intuition behind it, fun examples. It is effective in conveying the main ideas.

Benjamin states that his goal for the book was that the reader comes "away with an understanding of the major concepts, a better idea of why they work, and an appreciation for the elegance and relevance of each subject." While only the reader can decide if the author accomplished the last goal, Benjamin succeeds with regard to the first two. "The Magic of Math" is a great supplement to students learning mathematics in high school or college.

Gabriel Ngwe '17 is a math major from Chicago, Illinois. He enjoys chess, fencing, and mathematical analysis.